Rule of Universal Specification (US) (pg. 59): If a formula S results from a formula R by substituting a term t for every free occurrence of a variable v in R then S is derivable from $(\forall v)R$.

Def (flagged): A variable that is a free variable in a premise is called flagged. They are denoted by listing them when they are used in the right column (as in the deduction on pg. 61)

Rule of Universal Generalization (UG) (pg. 60): From a formula S we may derive $(\forall v)(S)$, provided the variable v is not flagged in S.

Def (ambiguous name): An "ambiguous name" (which is unfortunately not really *defined* in the book... but its nature is discussed through pages 80-85) is a "constant" and is ***not*** a "variable". We will use Greek letters $\alpha, \beta, \gamma, \delta, \epsilon$, etc to denote ambiguous names.

Rule of Existential Specification (ES) (pg. 83): If a formula S results from a formula R by substituting for every occurrence of a variable v in R an ambiguous name which has not previously been used in the derivation, then S is derivable from $(\exists v)R$.

Rule of Existential Generalization (EG) (pg. 83): If a formula S results from a formula R by substituting a variable v for every occurrence in R of some ambiguous name, then $(\exists v)S$ is derivable from R.

A question was raised in class about the use of rule E.G. Allow me to describe by first writing part of a deduction:

$\{1\}$	(1) $(\exists x)(Px)$	Premise
$\{1\}$	(2) $P\alpha$	1 ES

The question was whether or not the following would be a valid use of rule UG following the above:

$\{1\}$	(3) $(\forall x)(Px)$	2 UG

because clearly α is not a "flagged variable" and so it appears that UG could apply.

In fact it is not valid since the ambiguous name α is not a variable, which UG would require. Moreover if we had a premise in the argument with ambiguous name β of the form

$\{4\}$	(4) $R\beta$	Premise

we would not be listing β as a "flagged variable", again because it is not a variable! We must make this distinction because of the nature of our quantifiers $\forall$ and $\exists$:

- $\forall$ is the "universal quantifier" because it says "all objects in the universe have the property that..."
- $\exists$ is the "existential quantifier" because all that it says "there is at least one object in the universe such that..."

What happened above in line (3) above was that we converted the formula in line (1) involving an existential quantifier into a formula involving the universal quantifier. If this were possible, we would be entirely erasing the distinction between the two quantifiers!